Orthotropic Properties of Trabecular Bone Determined by Adaptation Models

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Introduction

Bone adaptation algorithms that predict the relationship between bone loads and density assume that bone is effectively isotropic. The isotropic assumption cannot explain bone anisotropy and directionality, which are evident from the trabecular architecture. We have modeled the trabecular bone in the proximal femur as effectively orthotropic. Instead of determining the density distribution, we used the local material directions and effective moduli as the variables determined by an adaptation algorithm. The algorithm proposed determines the local material directions and the elastic moduli in material directions, for each element of a 2D model of the proximal femur. Since the directions of the trabeculae correspond to material axes, this approach can explain the trabecular morphology and take bone anisotropy into account in modeling bone adaptation.

Methods

An adaptation algorithm was devised, where each iteration of the algorithm comprises of two stages. The first stage is to determine local material orientations, and the second stage is to determine the elastic moduli in material directions.

The orientation criterion utilized takes the multiple load cases applied to the femur into account. Orientations of the bone material were determined by rotating each finite element’s material axes to the directions of the largest principal stresses that occur due to multiple loads applied to the femur. We denote by $\sigma_{I(i,j)}$ the numerically largest principal stress for each element (i) and load case (j). The corresponding principal direction is denoted as $\theta_{I(i,j)}$. Absolute values of the principal stresses were used, assuming tension and compression have the same influence. The orientation $\theta_{(i)}$ of each element’s material axes in each iteration is determined by the angle

$$\theta_{(i)} = \theta_{I(i,p)}$$

(1)

where p is the load case that corresponds to the numerically largest principal stress:

$$|\sigma_{I(i,p)}| \geq |\sigma_{I(i,j)}| \quad j = 1...nlc$$

(2)

After this is determined for each element, the local material axes of each element are rotated accordingly.

The second stage is to determine the elastic moduli ($E_1$, $E_2$) in material directions (1,2) for each element. The purpose of this stage is to reinforce the local material stiffness in the dominant load direction, and allow a lower stiffness in other directions, assuming that the trabeculae can adapt differently according to their orientations. For this purpose two hypotheses were considered.

First, an existing isotropic ‘stimulus based’ adaptation algorithm was extended to include directional stimuli. For each element, two directional stimuli were defined, in each material direction. The stimulus defined is similar to the stimulus used by Beaupré et al. (1990), but it is based on the normal stresses in the material directions of each element, instead of a measure of SED. The elastic moduli are modified such that the directional stimuli tended to a constant reference value in both material directions of each element. The shear modulus of each element is estimated from the average of $E_1$ and $E_2$, and a constant Poisson ratio assumed. The algorithm proceeds until convergence is obtained.
Figure 1: (a) 2D Model of proximal femur utilized. Model comprises of inner trabecular elements, that are orthotropic, and a cortical shell, that participates in load bearing, but its adaptation is not considered. Eight distributed load cases were applied consecutively at each iteration. (b) Predicted elastic moduli ($E_1$, $E_2$) and material axes after convergence. Arrow lengths represent magnitude of moduli in each direction. (c) Predicted moduli as a function of density – comparison to experimental data. (d) Trabecular pattern predicted by ‘connecting’ the material axes predicted in Fig. 1b.
The second hypothesis used to determine $E_1$, $E_2$ is that trabecular bone adapts its local internal structure and effective moduli such that the maximal strains in each local material direction, due to all load cases, tend to a constant allowed strain. This is based on recent experiments that have shown that yield strains in trabecular bone are approximately isotropic and homogeneous (e.g., Chang et al., 1999). This approach is effectively a ‘Fully Strained Design’, similar to the ‘Fully Stressed Design’, which is an optimality criteria used in the field of structural optimization.

The algorithm was implemented on a 2D model of the proximal femur, shown in Fig. 1(a). The model differentiates between cortical and trabecular bone elements. The purpose of the model is to show the internal adaptation of the trabecular bone only. Hence the cortical bone appears as an exterior shell, which participates in load bearing, but is not part of the adaptation process in the algorithm.

**Results & Discussion**

The primary results of the algorithm are the local elastic moduli $E_1$, $E_2$ in material directions for each element, as shown in Fig. 1b. These results are for the stimulus-based hypothesis. The arrow directions are the directions of the local material axes, which are the predicted directions of the trabeculae in that region. The magnitudes of the arrows represent the Young’s modulus in the corresponding directions. A legend arrow is shown in the bottom right corner, with its magnitude representing a Young’s modulus of 2.5 Gpa.

In order to compare the results of $E_1$, $E_2$ with density distribution data, the density of each element was estimated using the average of $E_1$, $E_2$ and an empirical correlation between isotropic $E$ and density. This yielded a density distribution that conforms to the density distribution of the real bone (not shown). In order to compare with experimental results, the predicted values of $E_1$, $E_2$ were compared to experimental data relating to the trabecular bone in the proximal femur. According to Wirtz et al. (2000), the axial and transverse Young’s modulus are $E_1 = 1904\rho^{1.64}$ and $E_2 = 1157\rho^{1.78}$ respectively. The results of $E_1$, $E_2$ are compared in Fig. 1c with this data. As evident, over a large range of densities the prediction of the model is quite fair.

The predicted directions of material axes in Fig. 1b, that represent trabecular directions, can be seen to conform to the recognized directions of the trabecular pattern. To illustrate this, a post-processing program was written that ‘connects’ the directions of the element material axes, i.e., it connects the arrows into continuous lines. This is based on the notion that the trabeculae are approximately an orthogonal lattice. The result, shown in Fig. 1d, resembles the trabecular structure of the femur quite well. Results obtained with the ‘fully strained’ criteria were not different substantially, and it was hard to determine which hypothesis yields better results regarding the orthotropic moduli.

It is concluded that the results of the proposed algorithm are more informative than the common density distribution result, since material directions and the elastic moduli in material directions are predicted. Anisotropic representations of bone will enable to model implant-bone interaction in a correct manner, in order to improve stress-shielding prediction.

**References**