Evaluation of the yield criteria and the related scale effects of a synthetic foam model for trabecular bone under triaxial loading

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Introduction

A clear understanding of the multiaxial failure mechanism of trabecular bone is necessary to improve the prevention of osteoporotic bone fractures. The development and validation of a multiaxial testing technique for human trabecular bone is therefore required. Synthetic foams have been used as a model of trabecular bone to characterize new testing techniques independently of inter-specimen variability (Hale et al., 1999). Furthermore, rigid foams enable investigating the influence of specimen size in the multiaxial behavior of porous materials. The objective of this preliminary study was to evaluate various yield criteria and the associated size effects of a synthetic foam while subjected to triaxial loading in both a standard soil-mechanics testing system and a custom testing chamber designed for human trabecular bone samples.

Methods

A rigid polyurethane foam of 0.12 g/cc and 0.5 to 2.5 mm pore size was chosen to match the morphology and the mechanical behavior of osteoporotic trabecular bone. Twelve cylindrical samples of 100 mm length and 50 mm diameter were loaded to monotonic failure in a standard triaxial testing system (GDSMC Type 38-50mm/7kN/1.7MPa). The samples were subjected to either uniaxial compression, hydrostatic or axisymmetric loads with axial stress:radial pressure ratios of two and five at a stress rate of 0.2 MPa/hr. The resulting four yield points in the compressive quadrant of stress space were fitted to an isotropic Tsai-Wu criterion (Tsai and Wu, 1971). Three cylindrical samples of 100 mm length and 32 mm diameter waisted core were loaded to monotonic failure in a servo-hydraulic testing machine (MTS 809 Axial/Torsional Test System) at a strain rate of 0.0006 s⁻¹. In contrast to the specimens tested in the standard triaxial cell, these samples were gripped to aluminium end-caps to avoid structural end effects (Aspden, 1990; Odgaard et al., 1989; Zhu et al., 1994). At last, six cylindrical specimens of 10 mm length and 8 mm diameter of the same material were subjected to monotonic uniaxial compression in a custom loading chamber at a stress rate of 0.2 MPa/hr. The custom loading chamber was integrated to the servo-hydraulic testing machine, using the torsion channel to control the axisymmetric compressive stresses. These samples were also glued to end-caps to minimize end-artifacts. Axial force, axial displacement and radial pressure if applied, were measured by the built-in load-cell, LVDT and pressure transducers of each load frame. For all tests, the stress-strain curve in the axial direction was used to detect failure. The yield strains and stresses were defined using three different criteria: a residual norm of the applied stress tensor, a 10% and a 20% decrease of the slope of the axial stress-strain path. The ultimate stresses and corresponding strains for the waisted specimens loaded under displacement control were also computed.

Results & Discussion

In the standard triaxial system, the samples demonstrated homogeneous Young's moduli (26.5±0.6 MPa) and exhibited a ductile failure. Typical stress-strain curves presented an initial non-linearity, a well-recognized experimental artifact due to the interruption of the cellular structure at the machined surface (Keaveny et al., 1993; Zysset, 1994). In the servo-hydraulic testing machine, the waisted specimens exhibited likewise homogeneous Young's moduli (27.9±0.7 MPa) and a ductile failure. The initial non-linearity was eliminated by gluing the specimens to end-caps. The subtle difference in the mean Young’s moduli for the two sets of data was not statistically significant (p=0.06), suggesting that the distinct boundary condition of the two loading systems did not affect the elastic properties for the large samples. In the custom loading chamber, the smaller samples exhibited more scattered, significantly lower Young's moduli (16.5±3.2 MPa) and demonstrated a more fragile failure process. The reduced elastic moduli and the fragile failure
are attributed to the dominating effect of large pores in small samples. Further work will be done in order to quantify the variation of elastic properties as a function of specimen dimensions and boundary conditions.

The mean uniaxial elastic moduli and yield stresses using the three yield criteria for each testing system are presented in Table 1. Reliable results were found for the large samples tested both in the standard triaxial cell and the servo-hydraulic testing machine. The subtle differences in the mean uniaxial yield stresses for the two data sets using the 10% and 20% decrease of the slope criteria were not statistically significant (p=0.16 and p=0.15). However, the difference in the yield stresses using the residual norm of the applied stress tensor criterion was statistically significant (p=8.4E-6). In the custom loading chamber, the smaller samples exhibited more scattered, significantly different yield stresses for all yield criteria, being higher for the residual norm of the applied stress tensor criterion and lower for the other two criteria. This suggests a strong size effect in the yield properties of the synthetic foam too.

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Elastic</th>
<th>Yield</th>
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<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>E [MPa]</td>
<td></td>
<td>10% E [MPa]</td>
<td>20% E [MPa]</td>
</tr>
<tr>
<td>Standard triaxial cell</td>
<td>26.5±0.6</td>
<td>0.281±0.003</td>
<td>0.242±0.006</td>
<td>0.342±0.008</td>
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<tr>
<td>Servo-hydraulic testing machine</td>
<td>27.9±0.7</td>
<td>0.213±0.003</td>
<td>0.235±0.004</td>
<td>0.332±0.005</td>
</tr>
<tr>
<td>Custom triaxial cell</td>
<td>16.5±3.2</td>
<td>0.371±0.041</td>
<td>0.218±0.042</td>
<td>0.309±0.059</td>
</tr>
</tbody>
</table>

Table 1 - Elastic and yield properties computed in uniaxial compression for the three load frames.

From the multiaxial data acquired with the standard triaxial cell, the shape of the predicted Tsai-Wu criterion (1) in the axial stress-radial pressure space for each yield criteria was illustrated in Figure 1,

\[
\beta \cdot \sigma_{II} + \gamma \cdot \sigma_{I}^{2} = 1
\]

(1)

where \( \sigma_{I} \) and \( \sigma_{II} \) are the first, respectively the second stress invariant.

Figure 1 - Prediction of the Tsai-Wu model in the axial-radial stress space for the three yield criteria.

The predicted elliptical failure surfaces were validated only within the axisymmetric compressive stress quadrant. Its symmetric extension to the tensile stress space is a merely simplification since no tensile data
was available at this point. The two parameters (β and γ) associated to the quadratic terms of the Tsai-Wu model were highly sensitive to the selected yield criteria (Table 2). The adjusted coefficients of determination $r^2$ for the three yield criteria were respectively 0.978, 0.919 and 0.917.

<table>
<thead>
<tr>
<th>Yield Criteria</th>
<th>β [1/MPa$^2$]</th>
<th>γ [1/MPa$^2$]</th>
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<tr>
<td></td>
<td>-31.87</td>
<td>12.19</td>
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<tr>
<td>10% E</td>
<td>-43.58</td>
<td>17.90</td>
</tr>
<tr>
<td>20% E</td>
<td>-22.11</td>
<td>9.04</td>
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Table 2 - Experimental parameters of the isotropic Tsai-Wu criterion for the standard triaxial system.

A dramatic size effect for the uniaxial elastic and yield properties of the foam samples was observed, which implies that the influence of specimen size should be accounted for in the interpretation of uniaxial and multiaxial testing of human bone specimens. The high sensitivity of the yield definition in both uniaxial and multiaxial data sets questioned the use of these yield criteria when no unloading sequence is applied. Future tests in the custom loading chamber will be carried out using displacement control, and ultimate failure properties will be defined as the maximal stress and corresponding strain in the axial stress-strain graph.

With pore size up to 2.5 mm and the reported mechanical properties, the chosen polyurethane foam represents a valuable model for evaluation of size effects in mechanical testing of osteoporotic trabecular bone. A simple analytical model need to be developed to account for these effects. The collected results represent the basis for interpretation of human trabecular bone data obtained with our custom triaxial loading chamber.

References

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