Poroelastic Finite Element Model of Lumbar Spine in Sagittal and Lateral Moments Including Electrokinetic Coupling

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Introduction

During various functional activities, the motion segments of the lumbar spine undergo complex loadings consisting of forces and moments of different magnitudes and directions. Changes from a normal mechanical behavior in the spinal motion segment may cause the pathogenesis of herniation and degeneration of the intervertebral disc, end plate fracture, spinal deformity, low back pain syndromes, etc. The understanding of mechanical behaviour in SMSs is important in relation to the clinical problems occurring in the spine, clarification of the behaviour through a proper numerical models is also a great challenge for engineers. In this study we demonstrate differences between the solutions when the electrokinetical coupling is included.

Methods

A three-dimensional poroelastic finite element model of the spinal motion segments with the electrokinetic coupling is presented. The model includes the vertebrae (L5-S1) and the intervertebral disc. The governing equations consist of the conservation of both solid and fluid linear momentum and the fluid continuity condition. We use the interpretation of the swelling pressure effects through the electrokinetic coupling. It was found that the following relation can be established (which encompasses Ohm's and Darcy's laws as special cases):

\[
\begin{align*}
\{ \mathbf{q} \} &= -\left[ -k_{11} & k_{12} \right] \left\{ \nabla \mathbf{p} \right\} \\
\{ \mathbf{j} \} &= \left[ k_{21} & -k_{22} \right] \left\{ \nabla \phi \right\} 
\end{align*}
\]

where $\phi$ is the electrical potential, $k_{11}$ is the (short-circuit) Darcy's hydraulic permeability, $k_{22}$ is the electrical conductivity, and $k_{12}$ and $k_{21}$ are the electrokinetic coupling coefficients that are mutually equal according to the Onsager reciprocity (Frank et al., 1987).

We also take into account the large deformation of the solid by using the logarithmic strains and change of porosity during the deformations of the mixture. The resulting FE system of equations is presented here as

\[
\begin{align*}
\begin{bmatrix}
\mathbf{m}_{uu} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & \phi
\end{bmatrix}
\begin{bmatrix}
\Delta \mathbf{u} \\
\Delta \mathbf{p} \\
\Delta \mathbf{q} \\
\Delta \phi
\end{bmatrix} &= \begin{bmatrix}
\mathbf{k}_{uu} & \mathbf{k}_{wp} & 0 & 0 \\
0 & 0 & \mathbf{k}_{pq} & 0 \\
0 & \mathbf{k}_{wp} & \mathbf{k}_{qq} & \mathbf{k}_{qq} \\
0 & 0 & \mathbf{k}_{wp} & \mathbf{k}_{qq}
\end{bmatrix}
\begin{bmatrix}
\Delta \mathbf{u} \\
\Delta \mathbf{p} \\
\Delta \mathbf{q} \\
\Delta \phi
\end{bmatrix} \\
\begin{bmatrix}
\mathbf{m}_{uu} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & \phi
\end{bmatrix}
\begin{bmatrix}
\Delta \mathbf{u} \\
\Delta \mathbf{p} \\
\Delta \mathbf{q} \\
\Delta \phi
\end{bmatrix} &= \begin{bmatrix}
\mathbf{f}_u \\
\mathbf{f}_p \\
\mathbf{f}_q \\
\mathbf{f}_\phi
\end{bmatrix}
\end{align*}
\]

Here the nodal point variables are: the displacements of solid $\mathbf{u}$, the relative velocities $\mathbf{q}$, the pressures $\mathbf{p}$ and the electrical potentials $\phi$. The boundary conditions include: the general boundary conditions for the solid, the relative velocities, surface pressures, current density and the electrical potential (Kojic et al. 1998), (Filipovic, 1999).

Results & Discussion
The linear one-dimensional creep response (geometrical linearity) of the lumbar spine segment without and with electrokinetic coupling is shown in Fig. 1a (LINEAR and LINEAR-E), in order to demonstrate the effect of the electrokinetic coupling. The material constants for the electrokinetic coupling are taken from Frank et al. Obviously, the electrokinetic coupling (swelling pressure) plays an important role in the creep response of lumbar spine segment.

**Figure 1:** One dimensional model for creep response of lumbar spinal segment  
(a) Displacement of the top surface without (LINEAR) and with (LINEAR-E) electrokinetic coupling  
(c) Displacement of the top surface without and with electrokinetic coupling (LINEAR - geometrically linear, LOG-logarithmic strains; -E, with coupling)

Fig. 1b shows the FEM solution LOG-E (logarithmic strains, with electrokinetic coupling, the model FEM-2) that agrees well with the experiment (EXPERIM). If the constants corresponding to the analytical model are used in the large strain formulation and without electrokinetic coupling (LOG), or in the geometrically linear model (without and with electrokinetic coupling, LINEAR and LINEAR-E), the solutions differ significantly from the experimental results, as can be seen from the Fig. 1b, (Kojic et al. 2001).

**Figure 2a:** Mid-sagittal cross-section of the deformed segment under moment 60 Nm sagittal moment  
**Figure 2b:** Combined loading with sagittal and lateral moments under compression preload of 1000 N
Mid-sagittal cross section of the deformed segment under moment 60 Nm is shown in Fig. 2a. A combined loading with sagittal and lateral moments under compression preload of 1000 N is shown in Fig. 2b.

![Solid stress distribution of the lumbar motion segment under sagittal moment 60 Nm](image)

**Figure 3a:** Solid stress distribution of the lumbar motion segment under sagittal moment 60 Nm

**Figure 3b:** Variation of sagittal plane rotations with sagittal plane moments with and without EKC (electrokinetical coupling)

Solid stress distribution of the lumbar motion segment under sagittal moment 60 Nm is presented in Fig. 3a. Comparison between the predicted variation of the sagittal plane rotations in terms of the sagittal moments with and without electrokinetical coupling is shown in Fig. 3b, (Shirazi-Adl A. et al., 1998).

It can be noted that these additional effects give lower degree of rotation for the same sagittal moments. This study incorporate some essential features of the part of the lumbar spine segment: the accurate three-dimensional geometry, the geometric and material nonlinearities, variable porosity. Moreover, we have studied the effect of the swelling pressure represented through the electrokinetic coupling in case of coupled moments and rotations. The results demonstrate the important role of the electrokinetic coupling, nonlinear porosity, large logarithmic strains and the water content, on the very complex spinal response in prolonged loadings.

**References**