Modelling the nonlinear changes in body height due to external loading

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Introduction:
Changes in body height have been considered as a possible index to be employed for assessment of manual workload (Eklund et al., 1984; Tyrrell et al., 1985; Corlett et al., 1987; Van Dieen et al., 1993; Althoff et al., 1992; Leivseth et al., 1997) especially for cumulative loads from a prolonged working period with variation in physical working loads, as is common in many jobs involving manual materials handling. It is generally accepted that the time-dependent height changes mainly result from deformation of the spinal intervertebral discs. The intervertebral disc has viscoelastic characteristics since the material elastically responds to loading and unloading for short periods of time but continuously deforms, with decreasing strain rate if the load is kept constant for a long period of time.

Various types of mathematical models derived from combinations of spring(s) and dashpot(s) have been employed and developed to accurately represent behaviours of viscoelastic material, for example the Kelvin unit model, Kelvin solid model, Zener model and others including the Quasi-linear model. For the studies of changes in body height, all of the researchers mentioned above used linear viscoelastic models (Eklund et al., 1984; Van Dieen et al., 1993; Althoff et al., 1992; Leivseth et al., 1997). Although those models were able to give a good agreement with their experimental results from in vivo measurement of height change, they cannot be employed for representing and predicting changes in body height under variable loading. Therefore the objective of the present study was to investigate the possibility of modelling nonlinear changes in body height, both creep and recovery responses, under variable loading conditions.

Method:
To model nonlinear time-dependent changes in body height under variable loading, the general integral formulation of the constitutive equation of viscoelastic materials, as described in Findley et al. (1976), was used. The multiple integral form including the first three order terms is shown in Equation 1. By using the Heaviside unit function, Equation (1) can be integrated and written in the form of Equation 2 (a) for the first load step (0 < t < t₁) and of Equation 2(b) for the second load step (t > t₁), where loading is changed at time t₁.

\[
e(t) = \int_{0}^{t} \phi_1(t, \xi) \sigma(\xi) d\xi + \int_{0}^{t} \int_{0}^{t} \phi_2(t, \xi, t, \xi_2) \sigma(\xi) d\xi d\xi_2 + \int_{0}^{t} \int_{0}^{t} \int_{0}^{t} \phi_3(t, \xi, t, \xi_2, t, \xi_3) \sigma(\xi_2) d\xi d\xi_2 d\xi_3 + \cdots 
\]

\[
H(t) = R(t) \sigma_0 + M(t, t) \sigma_0^2 + N(t, t, t) \sigma_0^3 \quad \text{when } t < 0
\]

\[
H(t) = R(t) \sigma_0 + M(t, t) \sigma_0^2 + N(t, t, t) \sigma_0^3 + R(t-t_1) \sigma_0 + 2M(t, t-t_1) \sigma_0^2 + M(t-t_1, t-t_1) \sigma_0^3 + 3N(t, t-t_1, t-t_1) \sigma_0^3 + 3N(t, t-t_1, t-t_1) \sigma_0^3 \quad \text{when } t > t_1
\]

where \( H(t) \) is change in body height from height at \( t = 0 \)}
\( R(t), M(t,t) \) and \( N(t,t,t) \) are functions of time and material constants during the first load step (loading period) and \( R(t-t_1), M(t-t_1, t-t_1), N(t-t_1, t-t_1) \) and \( N(t-t_1, t-t_1, t-t_1) \) are functions for the second load step (recovery period).

Experimental data of change in body height under loading (from holding a barbell weighing 10, 20, 30 or 40 kg across the back of the subject’s shoulders) reported by Tyrrell et al. (1985) were employed to estimate the constant parameters of functions \( R, M, N \) in Equation (2a) for prediction of height loss during the loading period. The natural logarithm function in Equation (3) was considered for determining all parameter constants in the functions of time and material constants of Equation (2a) by substituting Equation (3) both in \( H(t) \) on the left hand side and in \( R(t), M(t, t), N(t, t, t) \) on the right hand side. Equation (2a), therefore, can be rewritten as Equation (4). \( H^0 \) and \( H^+ \) could be obtained from fitting the experimental data to Equation (3). The constant values (\( R^0, R^+, M^0, M^+, N^0, \) and \( N^+ \)) can be then determined by using Nonlinear Regression Analysis in SPSS (SPSS Inc., 2000) to fit Tyrrell et al.’s data to the relation illustrated in Equations (5) and (6). The exponential function in Equation (7) and the power function in Equation (8) were also used instead of Equation (3).

\[
H(t) = H^0 + H^+ \ln(t) \quad \ldots \ldots \ldots (3)
\]

\[
\frac{H^0}{\sigma_0} + \frac{H^+}{\sigma_0} \ln(t) = \left[ R^0 + M^0 \sigma_0 + N^0 \sigma_0^2 \right] + \left[ R^+ + M^+ \sigma_0 + N^+ \sigma_0^2 \right] \ln(t) \quad \ldots \ldots \ldots (4)
\]

\[
\frac{H^0}{\sigma_0} = R^0 + M^0 \sigma_0 + N^0 \sigma_0^2 \quad \ldots \ldots \ldots (5)
\]

\[
\frac{H^+}{\sigma_0} = R^+ + M^+ \sigma_0 + N^+ \sigma_0^2 \quad \ldots \ldots \ldots (6)
\]

\[
H(t) = H^0 + H^+ t^n \quad \ldots \ldots \ldots (7)
\]

\[
H(t) = H^0 - H^+ e^{-k t} \quad \ldots \ldots \ldots (8)
\]

Since the experimental data of changes in body height reported in Tyrrell et al. (1985) were not sufficient to calculate directly all of the constant parameters of Equation (2b) for prediction of height gain during the recovery period, it was therefore necessary to use an approximation to estimate the recovery response. Initially two approximation methods, the product form and the modified superposition principle (Findley et al., 1976), were tried but the closest fit was found by using Equation (2a) to estimate the recovery characteristic as well as the loading characteristics. The predicted values of changes in body height were compared with the experimental data by considering the average of the absolute % error (%ABS) as described in Burns et al. (1980).

**Results and Discussion:**

It was found that time-dependent changes in body height under variable loading could be modelled well by a nonlinear function of stress as shown in Figure 1. The degree of nonlinearity in function might be considered firstly from the isochronous graph of changes in body height per load, plotted as a function of the load in Figure 2. By fitting curves from Equations (5) and (6), the parameter constants of the nonlinear models were found to be significant in both the quadratic function and the cubic function of stress. Using the natural logarithm function of time and the exponential function of time gave a closer fit to the experimental data than using the power function of time. The % ABS ranged from 1.9% to 13.9%. However neither the product form or the modified superposition principle methods of approximation were found to give accurate results for predicting the body height during the recovery period, and the best approximation was found by fitting the first step equations of the general integral formulation separately to the loading and recovery phases of the in vivo records.
Further development to improving the accuracy of fit of these mathematical models for predicting changes in body height requires more experimental data of changes in body height under different loading conditions. In order to determine the parameters in Equation (2b), experimental data from multiple step loading (or strain) under different loads will be needed. It would be better and would permit the effects of posture to be considered if accurately estimated spinal loads could be used as input instead of the external loads carried, but these cannot at present be measured directly and only estimates of spinal loads can be found through the use of biomechanical models of the human body.

Figure 1 Changes in body height (solid points) and the predicted values (solid lines) during both loading and recovery periods, comparing two strain-time functions (a) the natural logarithm function and (b) the exponential function

Figure 2 Isochronous graphs illustrating direction of changes in body height per unit load during (a) loading period and (b) recovery period

References: