Curvature effects on mass transport in a human coronary artery bifurcation model

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Introduction
Disturbed mass transfer appears to have significant influence on blood vessel pathology. Therefore, the study of mass transport processes is important in the understanding of atherogenetic alterations. The transfer between the blood and the vessel wall affects the accumulation of potentially atherogenic molecules, such as ADP. A main aspect for realistic computer simulation of mass transport in arteries is the application of anatomically correct vessel geometries. In this respect the modeling of external coronary artery motion is of fundamental importance. This investigation applies a realistic model of the bifurcation of the left anterior descending coronary artery (LAD) and its first diagonal branch (D1) and physiologically correct flow conditions. The local curvature variation of coronary arteries on the beating heart causes large dynamic variations during the cardiac cycle. The aim of the present study is to analyze the influence of artery curvature changes on coronary artery mass transport of oxygen and ADP.

Methods
The computational model of the LAD-D1 bifurcation was developed from a digitized arterial cast. The generation procedure of the corresponding computational model has been explained in Perktold et al. (1998). Experimental curvature data of the heart surface measured at the site of the LAD-D1 branching were used to obtain curvature dynamics of the vessel. The mathematical model to describe the mass transport which is coupled to the blood flow in a time-dependent geometry employs the arbitrary Lagrangian-Eulerian (ALE)-modified convection-diffusion equation

\[
\frac{\partial c}{\partial t} + (u_i - \hat{u}_i) \frac{\partial c}{\partial x_i} - D \frac{\partial}{\partial x_i} \left( \frac{\partial c}{\partial x_i} \right) = 0 \quad i = 1, 2, 3
\]

where \( u_i, \quad i = 1, 2, 3, \) are the components of the flow velocity and \( \hat{u}_i, \quad i = 1, 2, 3, \) denote the components of the domain velocity; \( c \) is the concentration of the considered dissolved gas or macromolecule and \( D \) is the constant diffusion coefficient. The mathematical description of the blood flow uses the ALE-modified time-dependent, three-dimensional, incompressible Navier-Stokes equations for Newtonian fluids. The boundary condition at the artery wall depends on the considered molecules. For the transfer of dissolved gases (oxygen) a Dirichlet boundary condition (Back et al., 1977) is applied at the artery wall

\[
c_w = c_0 / 3
\]

where \( c_w \) is the wall concentration and \( c_0 \) is the reference concentration. For macromolecules an appropriate description uses the diffusive flux condition

\[
-n_i \cdot D \frac{\partial c_w}{\partial x_i} = P(c_w - c_I) \quad i = 1, 2, 3
\]

where \( n_i, \quad i = 1, 2, 3, \) denotes the components of the outward pointing unit normal vector at the boundary surface; \( P \) is the endothelial permeability and \( c_I \) is the concentration in the subendothelial
intima. The numerical solution uses the finite element Galerkin method. Due to the large Peclet number a streamline upwind Petrov-Galerkin stabilizing procedure and a subelement technique are applied (Rappitsch et al., 1996).

Results & Discussion

The study shows a significant influence of geometrical factors as bifurcation and curvature as well as vessel dynamics onto the concentration of the considered molecules which results in a variation of flux into the arterial wall. The results indicate a significant difference in wall flux between the myocardial and the pericardial wall. At decreasing curvature the wall flux gets significantly higher at the myocardial wall downstream of the bifurcation (Fig. 1:B). The results demonstrate that near the bifurcation the influence of the branching effect is significant. Downstream of the bifurcation the influence of curvature and curvature dynamics become dominant.

![Figure 1: Left: ADP wall flux (Sherwood number): A) During increasing curvature; B) During decreasing curvature. Top right: Flow pulse wave form in the LAD. Bottom right: Dynamic curvature during the pulse cycle.](image)

References


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