Trabecular Bone Adaptation Based on Tissue Level Stresses: A Cellular Solid Approach

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Introduction

Bone adaptation algorithms are finite element based computational models that simulate bone adaptation to mechanical loads. Existing algorithms model the trabecular bone at the ‘effective’ material level, where each finite element represents the effective properties of the underlying trabecular bone. Thus algorithms relate apparent density and effective material properties to effective stresses or a stimulus based on these stresses. There is no real relation to tissue level stresses, although it is quite clear that they are the main factor that determines the mechanical adaptation of trabecular bone. Some adaptation models published are at the tissue level (e.g., Mullender and Huiskes, 1995) but these present the adaptation of a small bone cube, and not of an entire bone. In order to base trabecular bone adaptation on tissue level stresses, a micro-structural model of trabecular bone is proposed, which links the continuum level and tissue level parameters. An orthogonal lattice is proposed, and it is used both to predict the effective material properties, and the tissue level stresses. We hypothesize that trabecular bone adaptation is driven by a tendency to adapt the internal architecture such that maximum tissue level stresses are equal in each element of the model. The hypothesis is implemented by a 2D orthotropic algorithm that modifies local material orientations and geometrical dimensions at the micro level in each effective element of the model, according to calculated tissue level stresses, based on multiple load cases applied to the bone.

Methods

The basic assumption of the model is that the mechanical behaviour of each finite element is governed by the mechanics of a representative unit cell, which is a simplified representation of the underlying trabecular bone. The cell geometry determines both the effective level properties of each element, and also the tissue level stresses as a function of the effective level stresses. For this purpose, an orthogonal lattice is utilized, with tubular struts and a rounded fillet joining struts (Fig. 1). Each representative cell (assumed to have a constant side length L) is described by its geometric parameters, the strut diameters \( t_1L, t_2L \) and an angular orientation \( \theta \) of the material axes. The main point of the model is the predicted deformation mode of the cell walls, depending on the type and direction of the local effective stresses. A normal stress directed in a material (trabecular) direction is assumed to deform it axially. A shear stress or an off-axis normal stress is assumed to cause bending deformation of the trabecular strut. These assumptions are consistent with Wolff’s trajectorial theory. A fillet was added at the rigid connection of the struts. The fillet increases the effective G, and reduces maximum bending stresses near the lattice joint.

The representative cell is analysed according to methods used in the field of cellular materials (e.g. Gibson, 1985). The effective level moduli are derived based on the assumption mentioned. The axial moduli are based on assumed axial deformation of the trabeculae. The shear modulus is derived based on bending deformations, using standard beam theory, assuming that the trabeculae bend with rotation constraints at their ends. These assumptions yield:
\[ E_1 = \frac{\pi}{4} E_{\text{tiss}} t_1^4 \]
\[ E_2 = \frac{\pi}{4} E_{\text{tiss}} t_2^4 \]  
(1)

\[ G_{12} = \frac{3\pi}{16} \eta E_{\text{tiss}} \left( \frac{t_1^4 t_2^4}{t_1^4 + t_2^4} \right) \]  
(2)

where: \( E_1, E_2 \) are the elastic moduli in material (lattice) directions 1,2; \( E_{\text{tiss}} \) is an ‘effective tissue modulus’ of 5 GPa (Kabel et al. 1999); \( G_{12} \) is the shear modulus, and \( \eta \) is a correction factor that takes the fillet into account (calculated numerically). A constant Poisson ratio \( \nu_{12} \) is assumed. The values obtained were compared to the experimental data in Wirtz et al. (2000), and showed a good agreement between the predicted moduli and experimental data.

The tissue level stresses were calculated from the effective level stresses, based on the same assumptions. Thus in each unit cell, at the tissue level:

\[ \sigma_{\text{max}} = \sigma_{\text{axial}} + \sigma_{\text{bending}} \]  
(3)

The microstructural model was incorporated into an iterative adaptation algorithm that modifies local lattice geometry in order to achieve constant maximal tissue level stresses, in a multiple load environment. Each iteration of the algorithm comprises of two stages. The first stage is to determine local material orientations. The orientations of each element were determined based on the assumption that the material axes, which correspond to the lattice directions, are oriented according to the direction of the largest principal stresses, at the effective level, that occur due to multiple loads applied to the femur. The second stage is modifying the diameters of the struts in order to achieve the mentioned goal. Tissue level stresses were calculated from effective level stresses obtained from the FE solution. In each element that the maximal tissue stress exceeds an allowed stress the corresponding trabecular strut diameter was increased, and vice versa. This process simulates bone deposition and resorption on the outer surface of the trabeculae.

**Figure 1:** Proposed microstructural lattice used to represent trabecular bone at the tissue level. Struts are tubular, with a rounded fillet connecting them. Each finite element is assumed to have the same mechanical properties as the underlying lattice. The lattice direction (orientation) is determined separately for each finite element.
Results & Discussion

Results of the algorithm are spatial distributions of the lattice orientations and geometric properties of the lattice representing each effective finite element. These are translated to effective level properties: apparent density, effective elastic moduli in material directions and the shear modulus of each element. Typical results are shown in Fig. 2. Material orientations predicted correspond very well with the known trabecular pattern in the proximal femur. The density distribution conforms fairly well with real bone, and the predicted orthotropic moduli were correlated to experimental evidence. A prediction of the degree of anisotropy is also made.

The proposed lattice model yields a two-fold advantage: it relates directly to tissue level stresses, and it is orthotropic, so it can explain the directionality of the bone architecture. The results suggest that the model presented, although being a simplified representation, can explain trabecular bone adaptation. This implies that trabecular bone adaptation might be driven by a tendency to adapt the internal architecture such that maximum tissue level stresses are homogeneous.

![Figure 2: Predicted density distribution and predicted elastic moduli (E_1, E_2) in material directions.](image)

References