CONSTITUTIVE MODEL FOR BEHAVIOUR OF BASEBALL

Rochelle L. Nicholls¹, Karol Miller¹*, Bruce C. Elliott¹
¹ The University of Western Australia, Perth, Western Australia
kmiller@mech.uwa.edu.au

INTRODUCTION

Regulating baseball response to impact is may be used to control the velocity of balls hit into the infield in baseball. This is necessary to reduce catastrophic impact injuries to defensive players. The baseball may deform to 50% of its original diameter during impact with the bat (Adair, 1997). In this paper, a large-deformation linear viscoelastic constitutive model for baseball time-dependent behaviour during unconfined compression is presented, based on quasi-static and high-speed impact experiments.

METHODS

To model time-dependent behaviour, shear coefficients, G, were expressed as an exponential series:

\[
G(t) = G_0 \left[ \alpha_{\infty} \exp \left( -\frac{t}{\tau_\infty} \right) + \sum_{i=1}^{n} \alpha_i G_i \exp \left( -\frac{t}{\tau_i} \right) \right]
\]

…1.

where \( \alpha_{\infty} \) and \( G_i \) are shear elastic modulii, \( \tau_i \) is the relaxation time for each Prony component, and \( \alpha_i \) is the relative shear modulus and \( G_0 = G_{\infty} + \sum_{i=1}^{n} G_i \)

\]

…2.

Finite deformation rate effects were taken into account through linear viscoelasticity by a convolution integral:

\[
\sigma_{\eta} = \int_0^t 2G(t-\tau) \frac{\partial e}{\partial \tau} d\tau
\]

where \( \sigma \) = Cauchy stress, \( e \) = deviatoric part of the strain, \( G(t) \) = shear relaxation function represented by Prony series:

\[
G(t) = \alpha_{\infty} + \sum_{m=1}^{N} \alpha_m \exp \left( -\beta t \right)
\]

…3.

Material coefficients were estimated through quasi-static uniaxial compression experiments, and verified by comparison with high-speed impact tests (Hendee et al., 1998). Unconfined uniaxial compression testing of seventy baseballs was conducted using the Instron 8501 hydraulic actuator. Testing was conducted at 1 mm.s\(^{-1}\), and terminated at 50% of ball diameter (35.8 mm). Previous research using tests to 10% ball diameter indicate the force-displacement relationship is linear (Hendee et al., 1998). From our data, the force-displacement curves are clearly nonlinear (Fig.1). Hence implicit finite element analysis (FE) was employed to evaluate the material response of the baseball. ANSYS 6.1 software was used to replicate the experimental setup. The ball was modelled as a solid sphere (radius 36 mm), subdivided to a one-eighth section due to the symmetry of the structure, and meshed with 675 8-node hexahedral elements. The steel platen was modelled as a rigid body. Loading was applied via the nodes of the platen in the vertical direction, at 1 mm.s\(^{-1}\) to 18 mm. A linear viscoelastic material model was adopted for the ball based on eqn (4). This material was assumed both isotropic and nearly incompressible. Time-dependency of the material was input using values from fit to experimental data (Table 1).

<table>
<thead>
<tr>
<th>( G_0 ) (MPa)</th>
<th>( G_{\infty} ) (MPa)</th>
<th>( K ) (bulk modulus) (MPa)</th>
<th>( \beta ) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>32.2</td>
<td>5.5</td>
<td>2097</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 1: Material constants for baseball.

Vertical reaction force data plotted against experimental curves to evaluate the material parameters (Fig.1). The adoption of an Extreme Mooney material model resulted in a better fit, but as this material function was not available for explicit FE in LSDYNA, it could not be further implemented.

REFERENCES