SUMMARY
A coupled model of a 3-D soft tissue continuum and a 1-D transient blood flow network is presented in this article. The primary aim of the computational model is to investigate the blood flow in major arteries of the diabetic foot. It has been reported in literature that there is a five-fold increase in stiffness of the plantar soft tissue in diabetic feet compared to healthy ones. The increased stiffness results in higher hydrostatic pressure within the plantar soft tissue structures when loaded (ground reaction force). The hydrostatic pressure acts on the external surface of blood vessels and tend to reduce the flow cross-section area and hence the blood supply.

The soft tissue continuum was modeled as a tri-cubic Hermite finite element mesh representing all the muscles, skin and fat of the foot and treated as incompressible with heterogeneous, anisotropic properties. The deformed state of the soft tissue continuum due to the applied ground reaction force was obtained by solving the Cauchy equation using the Galerkin finite element method.

The geometry of the main arterial network in the foot was represented using a 1D Hermite cubic finite element (FE) mesh. The flow model consists of 1D Navier-Stokes equations and a non-linear constitutive equation to describe vessels’ radius - transmural pressure relationship. The transmural pressure was computed as the difference between the fluid and soft tissue hydrostatic pressure. Transient flow governing equations were solved using the McCormack finite difference method.

The geometry of both the soft tissue continuum and arterial network is anatomically-based and was developed using the data derived from visible human images. Simulation results compare hydrostatic pressure distribution in both diabetic and healthy foot and resulting arterial flows.

INTRODUCTION
The foot of a diabetic patient is prone to complications such as peripheral vascular disease (angiopathy) and impairment of nerve function (neuropathy). These complications may result in foot ulceration and eventually amputation of the foot. Vascular disease in the diabetic foot can be divided into three major categories: large vessel disease, small vessels disease involving the arteries and arterioles and micro-vascular disease [1]. While micro-angiopathy leads to minor amputations caused by patchy or small areas of gangrene, vascular disease in large vessels may result in extensive gangrene and amputation of the entire foot. Macro-angiopathy in the diabetic foot can be categorised into two main areas: (i) atherosclerotic occlusion that causes an enlargement of in one area of the arteries because of an obstruction (e.g. thrombus) and (ii) hardening or calcification of the arterial wall [2].

A number of studies have investigated blood circulation in the diabetic foot using clinical data [3-4] and simulation results obtained from computational models [2]. Clinical studies primarily employed non-invasive Doppler ultra-sonography to estimate peripheral circulation rates in diabetic neuropathic feet. Hahn et. Al [2] developed a lumped-parameter, multi-domain (solid-fluid) model to investigate the effect of plantar pressure on the diameter of the arteries in the presence of arterial and soft tissue changes.

The computational model developed in this study is based on a 3D soft tissue continuum model undergoing large deformations (finite elasticity) due to applied external load (ground reaction force) with 1D flow network of major arteries (both dorsal and plantar) embedded in it. Continuum or distributed-parameter approach takes spatial variation of the model variables into account and hence produces more accurate predictions than those from lumped-parameter models. Coupling between fluid and soft tissue was archived via the vessel wall constitutive equation that described the relationship between vessel radius and the transmural pressure. The latter is the difference between the fluid pressure and soft tissue hydrostatic pressure which arises due to the incompressibility of the soft tissue material.

METHODS
Model geometries: The Bioengineering modeling software CMISS [5] was used to manually segment the visible human (VH) images [6] and derive the necessary data for the construction of a 3-d model of the foot. A previously developed fitting algorithm [7] was employed to construct an anatomically-based 3-d FE model of the entire foot. Each spatial coordinate was interpolated using tri-cubic-Hermite basis functions. Skin, subcutaneous fat and muscles were combined and represented as a single soft tissue continuum (Figure 1a).
A cubic Hermite 1-d network of major plantar and dorsal arteries of the foot was also created by fitting the data derived from the same VH images. Furthermore, on each image, extra data points were identified to infer the vessel cross-section and hence to estimate the mean radius at the location. The mean reference radius of vessels was also fitted as a linear field along the 1D geometry. The fitted FE mesh of the flow network with the radius field is shown in Figure 1b.

Figure 1: (a) 3-d FE model of the foot (b) embedded 1-d network of arteries; hydrostatic pressure distribution of (c) healthy foot (d) diabetic foot

Computational models: The deformation of the soft tissue continuum in response to the applied load (ground reaction force) can be obtained by solving the static Cauchy equation,

\[ \frac{\partial \sigma_{ij}}{\partial x_j} + \rho \bar{b}_i = 0 : i,j = 1..3 \]  

(1)

where \( \sigma_{ij} \) are Cauchy stress tensor components, \( x_j \) are spatial coordinates, \( b_i \) are body force components and \( \rho \) is the soft tissue density.

The weak form of the above governing equation with contact constraints (ground and sole of the foot) was formulated using the method of weighted residuals and final FE model equations were obtained using the Galerkin FE method. The non-linear algebraic equations so derived were numerically solved using the Newton-Raphson method.

The blood flow in arteries was modeled as an incompressible, homogeneous, Newtonian fluid with axisymmetric laminar flow in a compliant tube. It was further assumed that radial and circumferential flows are negligible compared to the flow in the axial direction and the axial velocity has parabolic variation in the radial direction. The reader is referred to reference [8] for more details of the fluid flow model. The reduced form of the Navier-Stokes equations (continuity and momentum) can be written as,

\[ \frac{\partial \bar{r}}{\partial t} + \nabla \cdot (\bar{r} \bar{V}) + \frac{\bar{R}}{2} \frac{\partial \bar{V}}{\partial x} = 0 \]  

(2)

where \( \bar{V} \) is the mean profile of the flow, \( \bar{R} \) is the vessel radius, \( P \) is the fluid pressure and \( \rho_f \) and \( \nu \) are fluid density and kinematic viscosity respectively. \( \alpha \) is the velocity profile parameter. The velocity profile is given by,

\[ u = \left( \frac{\alpha + 2}{\gamma} \right) \sqrt{\frac{1 - \left( \frac{R}{R_0} \right)^2}{1 - \left( \frac{R}{R_b} \right)^2}} \]  

and \( \gamma = \frac{2 - \alpha}{\alpha - 1} \)  

(3)

where \( u \) is the axial velocity at \( r \). The constitutive equation of the artery wall which describes the mechanical characteristics (vessel radius–transmural pressure relationship) was defined as a cubic polynomial of radius ratio \( \left( \frac{R}{R_0} \right) \),

\[ P_t = (P - p) = a \left( \frac{R}{R_0} \right)^3 + b \left( \frac{R}{R_b} \right)^2 + c \left( \frac{R}{R_b} \right) \]  

(4)

where \( P_t \) is the transmural pressure (difference between fluid and solid hydrostatic pressures), \( R_0 \) is the reference radius and \( a, b \) & \( c \) are the polynomial fitting coefficients. The above system of equations was solved numerically using the McCormack’s finite difference method.

RESULTS AND DISCUSSION

Figures 1(c) and 1(d) depict the hydrostatic pressure distribution within the soft tissue continuum and resulting radii and flows in the arterial network for the healthy and diabetic foot respectively.

CONCLUSIONS

A coupled computation model consisting of a 3-d soft tissue continuum and a 1-d transient flow network was developed. The model was used to compare and investigate the deformation of the diabetic foot and resulting blood flow changes in the major arteries of the healthy and diabetic foot. The coupling of soft tissue and fluid was achieved via the vessel constitutive equation, which is a function of the soft tissue hydrostatic pressure and blood pressure.

REFERENCES