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Modeling and analysis of redundant musculoskeletal systems using null space projection

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INTRODUCTION

It is an undeniable fact that musculoskeletal systems are inherently redundant, i.e., there are more degrees of freedom than those required to perform certain tasks and each degree of freedom is actuated by multiple muscles. This over-availability poses numerous challenges in the process of modeling and simulation that can negatively affect the validity of the models and the obtained results, rendering their application frequently inappropriate for clinical practice. The projection from a low- to a high-dimensional space is not uniquely defined (e.g., muscle sharing problem), however, this indeterminacy can be captured mathematically using the notion of null space. This study presents a method for calculating the feasible muscle forces that satisfy the movement and physiological muscle constraints. Its importance is demonstrated in the context of joint reaction and joint stiffness analysis. Results demonstrate that 1) misinterpretation of the results is possible if the redundant (null space) solutions are ignored and 2) the musculoskeletal system is capable of achieving a highly variable stiffness using muscle co-contraction, highlighting the importance of performing feasibility studies.

METHODS

The muscle forces $\boldsymbol{f}_m \in \mathbb{R}^m$ are related to the generalized forces $\boldsymbol{\tau} \in \mathbb{R}^n$ through the moment arm matrix $\boldsymbol{R}$. When the muscle forces are known, the generalized forces are calculated uniquely. Since the muscle forces cannot be measured non-invasively, they are estimated from experimentally measured kinematics and externally applied forces. However, this approach results in an underdetermined set of equations, which is typically solved by formulating an optimization problem that minimizes some objective criterion. A particular solution will not only bias the results, but will also affect the calculation of other quantities that depend on the muscle forces [1, 2]. Therefore, identification of the feasible solution space can help to properly interpret results obtained from the redundant musculoskeletal systems.

In a typical experimental setup the motion and externally applied forces are recorded. Given these recordings, inverse kinematics and inverse dynamics are performed in order to assess the model kinematics and kinetics that satisfy the experimental measurements. Instead of estimating the muscle forces by forming an optimization problem, one could find the family of solutions that satisfy the following equation for the known $\boldsymbol{\tau}$

$$\boldsymbol{f}_m = -\boldsymbol{R}^{+T}\boldsymbol{\tau} + \boldsymbol{N_R}\boldsymbol{f}_{m0}, \boldsymbol{N_R} = \boldsymbol{I} - \boldsymbol{R}\boldsymbol{R}^{+} \qquad (1)$$

where $\boldsymbol{N_R}$ represents the null space moment arm matrix and $\boldsymbol{f}_{m0}$ an arbitrarily selected vector. Note that $\boldsymbol{f}_m$ spans the entire $\mathbb{R}^m$ for some arbitrary value of $\boldsymbol{\tau}$ and $\boldsymbol{f}_{m0}$, whereas in reality muscle forces are strictly positive (contraction) and bounded (limited force). However, the null space term can provide a suitable correction in order to satisfy the physiological muscle constraints. Assuming a linear muscle model

$$\boldsymbol{f}_m = \boldsymbol{f}_{max} \circ \boldsymbol{a}_m, \boldsymbol{0} \preccurlyeq \boldsymbol{a}_m \preccurlyeq \boldsymbol{1} \qquad (2)$$

where $\boldsymbol{f}_{max}$ denotes the muscle strength and $\boldsymbol{a}_m$ the muscle activation, one could observe that Eqs. (1) and (2) must be equal

$$-\boldsymbol{R}^{+T}\boldsymbol{\tau} + \boldsymbol{N_R}\boldsymbol{f}_{m0} = \boldsymbol{f}_{max} \circ \boldsymbol{a}_m, \boldsymbol{0} \preccurlyeq \boldsymbol{a}_m \preccurlyeq \boldsymbol{1}$$

$$\begin{bmatrix} -\boldsymbol{N_R} \\ \boldsymbol{N_R} \end{bmatrix} \boldsymbol{f}_{m0} \preccurlyeq \begin{bmatrix} -\boldsymbol{R}^{+T}\boldsymbol{\tau} \\ \boldsymbol{f}_{max} + \boldsymbol{R}^{+T}\boldsymbol{\tau} \end{bmatrix}. \qquad (3)$$

It can be shown that Eq. (3) is convex and bounded thus can be sampled for $\boldsymbol{f}_{m0}$ using vertex enumeration techniques [1]. Therefore, the feasible muscle forces can be obtained by adding the solutions that satisfy Eq. (3) to the particular solution

$$\{\boldsymbol{f}_m\} = \{-\boldsymbol{R}^{+T}\boldsymbol{\tau} + \boldsymbol{N_R}\boldsymbol{f}_{m0}^{i}, \forall i\}.$$

RESULTS AND DISCUSSION

Two case studies are presented, where the influence of musculoskeletal redundancy on the feasible joint reaction load and joint stiffness is evaluated. An anatomically realistic gait model (OpenSim [3]) is utilized. The model has ten degrees of freedom and eighteen muscles, features lower extremity joint definitions, low back joint and a planar knee model.

An accurate estimation of the muscle forces is essential for the assessment of joint reaction loads. The normalized (with respect to body weight) reaction forces on the knee joint during walking are reported in Fig. 1, along with the heel strike and toe-off events. The shaded area depicts the reaction force range as attributed to the null space solutions of muscle forces. The red dotted line represents the results obtained from OpenSim by minimizing the square of muscle actications. The large range of possible values confirms that misinterpretation of results is possible if the null space solutions are ignored. Consequently, the null space contributions can significantly alter the reaction loads without affecting the movement.

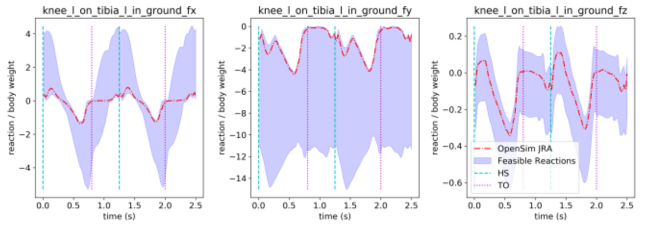

Fig. 1: Comparison between feasible reaction forces on the knee (proposed method) and optimization-based obtained (OpenSim).

Intuitively, stiffness (or rigidity) is the extent to which the limbs resist movement induced by external forces. It can be shown that the joint stiffness can be calculated from the muscle forces [2]

$$\boldsymbol{K}_j = -\frac{\partial \boldsymbol{R}^T}{\partial \boldsymbol{q}} \boldsymbol{f}_m - \boldsymbol{R}^T \boldsymbol{K}_m \boldsymbol{R} \qquad (4)$$

where the first term captures the varying effect of the muscle moment arm matrix, while the second term maps the muscle space stiffness $\boldsymbol{K}_m$ to joint space. In order to compute the joint space stiffness using Eq. (4), a model for selecting the muscle stiffness must be defined. The definition of short range stiffness [4]

$$k_{ii} = \gamma \frac{f_m}{l_m^o}$$

is used for calculating the diagonal elements (inter-muscle coupling is ignored) of $\boldsymbol{K}_m$, with $\gamma = 23.4$ and l_m^o being the optimal fiber length.

Fig. 2 depicts the feasible joint stiffness of the hip, knee and ankle joints during walking with the heel strike and toe-off events annotated accordingly. These results confirm experimental measurements and furthermore present similarities in the outline of the minimum stiffness predicted by our method. Notably, the hip stiffness range is gradually decreasing between heel strike and toe-off, because the flexor muscles are preparing for the swing phase and the capacity to increase the joint stiffness riches its lowest value before the toe-off event. A similar pattern is observed at the knee joint, which undergoes a flexion and a subsequent extension during the swing phase. We observe that the capacity of the muscles to modulate the ankle stiffness is not decreased and the range is gradually shifted upwards in the region between the heel strike and toe-off events. The increase in the minimum possible values of the ankle stiffness is attributed to the counterbalance of the ground reaction forces by the ankle plantar flexion muscles. As muscle effort is spent by these muscles, one would expect a lower maximum bound, which is not the case here. This could be contributed to the fact that the musculoskeletal system is asymmetric, i.e., the plantar flexion muscles can induce larger magnitudes of moment at the ankle joint in comparison to the dorsiflexion muscles. We can conclude that the contribution of the ground reaction forces results in an increase of the ankle stiffness.

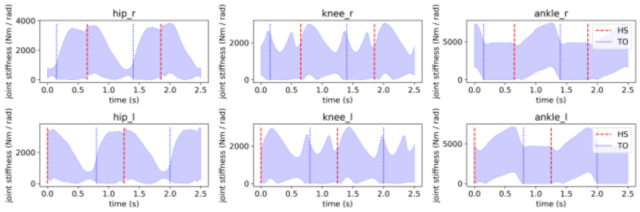

Fig. 2: The feasible joint stiffness of the hip, knee and ankle joints during walking with the heel strike (HS) and toe-off (TO) events annotated accordingly.

CONCLUSIONS

Null space solutions, although typically ignored in musculoskeletal system simulations, offer deep insights and provide a much broader framework for modelling, simulation and analysis of redundant musculoskeletal systems.

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