REASONABLE SIMULATIONS DEPEND ON RELATIONSHIPS BETWEEN CALCULATED RESIDUAL FORCES AND MOMENTS: IMPLICATIONS FOR RESIDUAL THRESHOLD LIMITS

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INTRODUCTION
Residual force and moment quantities assess the dynamic inconsistencies (i.e., errors) of a simulation [1]. Subjective recommendations for the maximum allowable residuals have been recommended for reasonable simulations of walking and running [2]. Though some recommendations have been purported in the literature, standards for residual thresholds, however, have not yet been established nor formalized. The use of simulation within the human and animal sciences has evolved beyond walking and running gait to include more dynamic tasks like landing, change of direction, etc. As such, there is need for researchers to establish more formal guidelines associated with what constitutes an acceptable or reasonable simulation for research purposes.

The purpose of this study was to assess the effects residual forces have on the predicted ground reaction forces (GRF), specifically the zero-moment point (ZMP) location, of a simulation. Additionally, we will propose a computational framework for establishing residual force thresholds for acceptable or reasonable simulations of human and animal movement.

METHODS
Zero moment point computations can be used to predict simulated GRF from the experimental kinematic measures for single stance phases of movement [3, 4]. Inverse dynamic computations first find the model’s residual forces \( \bar{F}_r \) and moments \( \bar{M}_r \) (at the pelvis whose position is \( r_p \) = \( \{X_p, Y_p, Z_p\} \) with respect to the ground origin in our case), computed without applied external loads. The residuals are then transformed to a point on the ground that would have an equivalent effect on the model. If errors in the these transformed forces and moments are zero, the resulting ZMP-estimated forces and moments are dynamically consistent with experimental GRF and center of pressure (COP) measures [4, 5]. As errors increase, the residuals beyond the experimental GRF increase, which can be quantified as changes in ZMP estimations.

\[
\bar{F}_{ZMP} = \bar{F}_p \\
\bar{M}_{ZMP} + \bar{r}_{ZMP} \times \bar{F}_{ZMP} = \bar{M}_p + \bar{r}_p \times \bar{F}_p
\]

(1)

\( F_{ZMP} \) and \( M_{ZMP} \) are forces and moments calculated at a point on the ground positioned at \( r_{ZMP} \) with respect to the ground origin (Eq. 1 & 2). Like calculating the COP, the ZMP location for a given position of the pelvis \( r_p \) can be calculated as follows:

\[
X_{ZMP} = f(\bar{M}_p, F_{X_p}, F_{Y_p}) = X_p + \frac{(\bar{M}_p - Y_p F_{X_p})}{F_{Y_p}} \tag{3}
\]

\[
Z_{ZMP} = g(M_p, F_{Z_p}, F_{Y_p}) = Z_p - \frac{(M_p + Y_p F_{Z_p})}{F_{Y_p}} \tag{4}
\]

From above (Eq. 3 & 4), the change in X (anteroposterior) and Z (mediolateral) positions of the ZMP are functions that describe the potential errors due to increased residuals as the total differentials in the three residual variables:

\[
\Delta X_{ZMP} = \frac{1}{F_{Y_p}} (\Delta M_p - Y_p \Delta F_{X_p} - \frac{(M_p - Y_p F_{X_p}) \Delta F_{Y_p}}{F_{Y_p}}) \tag{5}
\]

\[
\Delta Z_{ZMP} = \frac{1}{F_{Y_p}} (\Delta M_p - Y_p \Delta F_{Z_p} + \frac{(M_p + Y_p F_{Z_p}) \Delta F_{Y_p}}{F_{Y_p}}) \tag{6}
\]

The applied interpretation of the computational relationships (Eq. 1 – 6) prove that the changes in the position of the ZMP are reduced for movements with higher vertical forces. To better understand the residual relationships between horizontal residual forces and moments and the X and Z (transverse plane) position of the ZMP, we varied the \( F_{X_p}, F_{Z_p}, M_p \) and \( M_p \) residuals (\( F_{Y_p} \) residual not varied here for brevity). Residual forces were normalized by \( F_{Y_p} \) and \( Y_p \) and residual moments by \( F_{Y_p} \). ZMP changes were normalized by foot length (aligned in X direction) and the foot width (aligned in Z direction). Deviations of the ZMP location outside the foot (±50% of foot length and foot width) were not quantified as we assume this to mean that the ZMP deviated beyond the
boundaries of the foot or outside the model's base of support (BoS).

RESULTS AND DISCUSSION

ZMP changed proportional to a coupled relationship of paired residual force and moment (Fig. 1 & 2). Importantly, there were multiple values of coupled residuals in a predictable relationship that did not impart a change in ZMP, which represents a perfect agreement between the ZMP position and the experimental COP position (see red lines in figures below). Change in ZMP position was inversely proportional to vertical force ($F_{Y_p}$, not shown for brevity). Relationships with vertical force prove that more dynamic movements could afford to have higher residuals, with minimal sacrifice to the dynamic consistency of the simulation.

\[ R_{\text{ZMP}} = \frac{F_{Y_p}}{M_{Z_p}} \]

**Fig. 1:** $X_{\text{ZMP}}$ (anteroposterior position) changes with residuals $F_{X_p}$ (anteroposterior force) and $M_{Z_p}$ (moment about mediolateral axis). Red line indicates no change in ZMP location across multiple residuals.

**Fig. 2:** $Z_{\text{ZMP}}$ (mediolateral position) changes with residuals $F_{Z_p}$ (mediolateral force) and $M_{X_p}$ (moment about anteroposterior axis). Red line indicates no change in ZMP location across multiple residuals.

From Eq. 5 & 6, it is evident that increasing residuals in isolation changes the ZMP position, and ultimately causes the ZMP to go beyond the boundaries of the foot or outside the BoS. A relationship between residual $F_{X_p}$ (anteroposterior force) and residual $M_{Z_p}$ (moment about mediolateral direction) shows that an increase in one of these residuals accompanied by decrease in the other pushes the ZMP outside of the BoS in the anteroposterior ($X$) direction (Fig. 1). A similar relationship is observed for the ZMP mediolateral ($Z$) position (Fig. 2).

Importantly, the ZMP anteroposterior ($X$) position did not change when $F_{X_p}$ and $M_{Z_p}$ residuals simultaneously changed in the same direction with equal normalized magnitude. The ZMP mediolateral ($Z$) position also did not change when $F_{Z_p}$ and $M_{X_p}$ residuals simultaneously changed in opposite directions with equal normalized magnitudes. These unrecognized relationships result from the fact that the paired residuals ‘canceled each other out’ and show that it is possible to have high individual residual estimates, if, and only if, another residual is sufficiently high to ‘cancel out’ deleterious effects on the ZMP position. In other words, individual residual force and moment threshold magnitudes should not be considered independent from each other; alternatively, the relationship and coupled effects on the ZMP position and resulting dynamic consistency of a simulation should be considered together when defining boundaries or making recommendations related to residual force and moment thresholds for a reasonable simulation. It is also important to note that because the effect of high residual forces on ZMP position can ‘cancel out’, the time varying kinematics of a simulation should be considered in parallel when assessing the dynamic consistency of a simulation.

CONCLUSIONS

Dynamic movements with different velocity characteristics can and should have different residual force and moment thresholds. The inherent collinearities between individual force and moment residuals, which affect the dynamic consistency of a simulation (i.e., ZMP position versus COP), deserves future research to establish functional residual thresholds for simulations of human and animal movement.

REFERENCES