

STRUCTURAL MODELLING OF TRABECULAR BONE ADAPTATION USING A VORONOI NETWORK

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INTRODUCTION

Previously developed structural finite element models of trabecular bone adaptation [1-2] have used randomized networks of truss elements which consider axial force, requiring high nodal connectivity (the number of structural elements connecting to each node) to maintain structural and computational stability. With element level adaptation, allowing elements to reduce cross-sectional area or Young's modulus until their contribution is negligible, nodal connectivity can reduce but remains higher than that observed in bone samples [3]. This study presents the use of a Voronoi network of beam elements which consider axial force, biaxial bending and torsion moments, requiring lower nodal connectivity to maintain structural and computational stability, and allowing structural adaptation to be carried out at the element and network level.

The majority of studies seeking to model trabecular bone adaptation in response to mechanical stimulus have adopted a continuum finite element modelling (cFEM) approach, at either the macro-scale using elements larger than individual trabeculae [4], or at the micro-scale using elements smaller than individual trabeculae [5]. While cFEM can be effectively utilized to investigate bone modelling and remodelling at both scales, at macroscale it is not capable of elucidating possible structural forms for trabecular bone, and at microscale it is exceptionally computationally demanding.

A computationally efficient alternative to microscale cFEM is structural FEM (sFEM) in which the structure being modelled is represented as an idealized network of truss or beam elements rather than as a continuum of solid elements. For human vertebral bone samples [6] demonstrated excellent agreement between both FEM approaches, with a 1,000 to 10,000 fold reduction in computational time for sFEM models built using beam elements in comparison to microscale cFEM models, depending on the mesh resolution.

Studies that have used sFEM to model bone adaptation in the femur [1] and pelvis [2] in

response to multiple load cases have used truss elements, which deform in the axial direction but ignore bending and torsion. This is consistent with Wolff's 'law' that bone forms along trajectories associated with the directions of principal compressive or tensile stresses, minimizing bending. The resulting sFEM models have been successfully used to predict fracture loads and patterns in the femur [7].

However, the use of truss elements in bone adaptation results in structures with artificially high nodal connectivity (NC) as seen in optimized long-span space structures such as those used in airports, railway stations and conference centers. A minimum NC of 6 results due to the orthotropic nature of the principal stress directions that bone is believed to form along, while a higher NC is required to resist multiple load cases which introduce shear due to off axis loading compared to the principal stress directions found when adaptation is carried out for a single load case. Recent studies [3] contradict the trajectory hypothesis for bone adaptation, indicating frequent NC values of 3 and 4 with elements connecting into nodes in common structural arrangements or motifs. A Voronoi network provides an abundance of nodes with an initial NC of 4.

METHODS

This study presents preliminary results from virtual in-silico bone samples built using the Voronoi method of partitioning space around points, so that each edge in the resulting structure is equidistant from two or more points. 3D Voronoi networks are built in Rhino using the Grasshopper parametric design tool and analyzed using the Abaqus finite element solver.

The strain driven adaptation described in [1-2] is being extended to allow biaxial bending moment as well as axial forces to be considered in element level bone adaptation.

Fig 1 shows an example of a 25x25x25mm Voronoi network structure built from 100 random points, subject to compressive loading causing 20% strain in the vertical direction. Each

structural member running between two nodes is modelled using four beam elements with a Hermite cubic shape function. The radius of all elements is set to 0.25mm in the initial state.

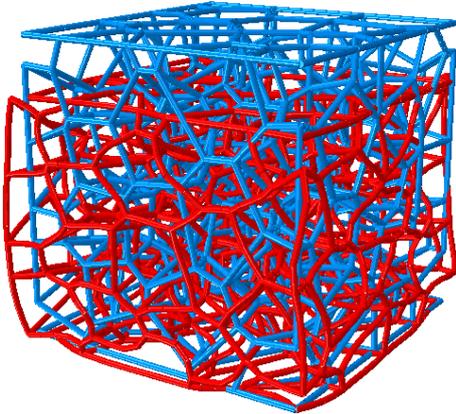


Fig 1: A Voronoi network structure subject to a 20% compressive strain in the vertical direction. Undeformed shape in blue, deformed shape in red.

RESULTS AND DISCUSSION

The axial force, biaxial bending and torsion moment distributions for the Voronoi network introduced in Fig 1 are shown in Figs 2 to 4.

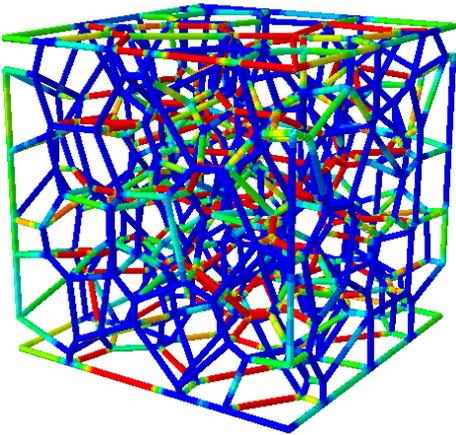


Fig 2: Axial force distribution shown on the undeformed shape. Rainbow shading, blue indicates compression ($\leq -10\text{N}$), red indicates tension ($\geq +10\text{N}$).

From Fig 2 it can be seen that those members close to vertical in orientation are subject to compression while those close to horizontal are subject to tension.

From Fig 3 it can be seen that in general bending moment is higher towards the nodes and lower towards the center of the structural members. This is in agreement with the observation that trabecular bone architecture is built up towards the nodes, with higher bending moment resulting in higher normal stress and strain at the extreme fibers of the structural members towards the nodes, which reduces proportional to the cube of the trabecular radius. Rapid change in bending moment close to the nodes may present a risk of micro-cracking in this zone, where trabecular radius also undergoes rapid change.

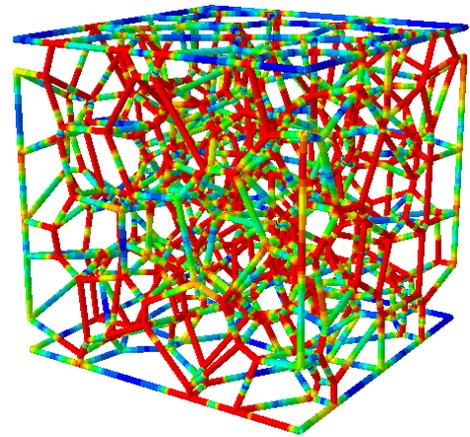


Fig 3: Biaxial bending moment magnitude distribution. Rainbow shading, blue indicates close to zero, red indicates $\geq 10\text{Nm}$.

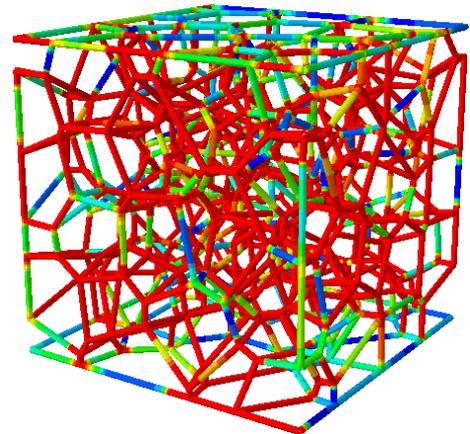


Fig 4: Torsion moment magnitude distribution. Rainbow shading, blue indicates close to zero, red indicates $\geq 1\text{Nm}$

From Fig 4 it can be seen that the majority of trabeculae are under torsion. While the values are less than those for bending moment they are sufficient to significantly alter principal stress and strain values and orientations within trabeculae and may account for specific micro and nano-structural arrangements.

CONCLUSIONS

Based on these preliminary results using a Voronoi network is considered a promising approach to model trabecular bone adaptation. Ongoing work will investigate the adaptation of trabecular bone through varying element cross-sectional properties and through the iterative placement of the points used to construct the Voronoi network to realign structural members based on the loading envelop that bone is subjected to during activities of daily living.

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