FAST AND ROBUST MUSCLE PATH WRAPPING FOR ARBITRARY SURFACES

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INTRODUCTION

Musculoskeletal models play an important role in biomechanical research, surgical planning and clinical assessment. For computational efficiency, they frequently employ simple cable-like muscle representations, which run between "origin" and "insertion" points while wrapping around obstacles such as bones. The resulting wrap path is important for correct simulation but not trivial to compute. Current biomechanical simulators typically restrict the obstacle geometry to analytic shapes such as cylinders, ellipsoids, and tori [1], or parametrically defined surfaces [2]. In this work, we propose a novel approach that works for arbitrary mesh-based surfaces (Figure 1) with low computational cost. This is useful because wrapping obstacles often correspond to anatomical structures such as bone and cartilage, with shapes that are derived from medical imaging.

METHODS

Our method works by subdividing the wrapable strand into a set of segments divided by knot points (Figure 2). Artificial elastic forces are then used to draw the knots together and keep them from penetrating obstacles. These forces are invisible to the simulation: the knots have no mass, and their forces are used to simply "shrink wrap" each segment around the obstacles to form a shortest-distance geodesic path, which is updated each simulation step to accommodate motion of the obstacles and origin/insertion points.

Assume that a wrapable segment has \( m \) knots, indexed by \( k = 1, \ldots, m \), each located at a position \( \mathbf{x}_k \).

The total force \( f_k \) acting on each knot is given by

\[
f_k = f_{wk} + \sum_c f_{ck},
\]

where \( f_{wk} \) is the \textit{wrapping force} and \( f_{ck} \) is the \textit{contact force} between the knot and obstacle \( c \). Wrapping forces are computed linearly with respect to the neighboring knots:

\[
f_{wk} = K_w (\mathbf{x}_{k+1} - 2\mathbf{x}_k + \mathbf{x}_{k-1}),
\]

where \( K_w \) is the \textit{wrapping stiffness}. To compute the contact forces \( f_{ck} \), we determine, for each obstacle, the knot's distance to the surface \( d_k \), where \( d_k < 0 \) implies penetration, together with the associated normal direction \( \mathbf{n}_k \). Then

\[
f_{ck} = \begin{cases} -K_c d_k \mathbf{n}_k & \text{if } d_k < 0 \\ 0 & \text{otherwise} \end{cases},
\]

where \( K_c \) is the \textit{contact stiffness}. For shapes such as cylinders, ellipsoids, and tori, \( d_k \) and \( \mathbf{n}_k \)
can be computed analytically, while for general mesh-based surfaces, they are interpolated using a signed distance grid generated from the mesh. Quadratic interpolation is used as it yields smoother results with less grid storage.

Letting $\mathbf{x}$ and $\mathbf{f}$ denote the aggregate position and force vectors for all knots, computing the wrap path involves finding the equilibrium position for which $\mathbf{f}(\mathbf{x})$ is close to zero. To accommodate the contact nonlinearities, this is solved iteratively using a damped Newton method, with the update at each iteration $j$ given by

$$\mathbf{x}^{j+1} = \mathbf{x}^j + \alpha(D\mathbf{I} - \mathbf{K})^{-1}\mathbf{f}(\mathbf{x}^j),$$

(1)

where $D$ is a constant damping parameter, $\alpha$ is an adaptively computed step size adjustment, and

$$\mathbf{K} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}$$

is the force derivative. Because $\mathbf{K}$ is a simple block-tridiagonal matrix, (1) can be computed quickly, in $O(m)$ time.

![Figure 3: A/B points on an obstacle surface, used to compute the reaction forces.](image)

Once a strand's path has been computed, it is employed in the simulation by first computing a strand tension $F(l, \dot{l}, \alpha)$, where $l$ is the overall strand length and $\alpha$ is the muscle excitation level, and then using $F$ to determine the reaction forces on each obstacle. It is sufficient to consider only the two points $A$ and $B$ at which the strand enters and leaves obstacle contact. In order to alleviate discretization errors as the knots move across the obstacles, these A/B points are determined by computing the points on the obstacle surface that are tangent to the ideal wrap path (Figure 3).

### RESULTS AND DISCUSSION

We have implemented our method and incorporated it into ArtiSynth (www.artisynth.org, [3]), an open source mechanical simulation platform combining multibody and finite element capabilities. Within the simulator, strand wraps paths are initialized at the start of the simulation and then updated at the end of each simulation time step.

Theoretical analysis shows that the stiffness parameters $K_w$ and $K_c$ are simulation independent and can usually be set at 1 and 10, respectively. Experimentally, we have found that a good damping value is typically $D = 10K_w/m^2$, where $m$ is the number of knots. The number of iterations required to solve (1) is frequently under 10, and in any case, full convergence is usually not required since the calculation is resumed each simulation step.

A number of experiments were performed to assess performance. First, solution accuracy was measured using test obstacles consisting of spheres and cylinders, for which exact analytic solutions can be computed, and comparing the exact results with those from our method. In these tests, the maximum error in the forces and moments on the obstacles was found to be under 1 percent. Secondly, dynamic response was evaluated by allowing a cylinder to drop and bounce off a double strand with different tensions $F$. The resulting period had an average error of 0.34 percent compared to the expected theoretical value. Finally, wrapping was tested on a number of general mesh surfaces (such as those in Figure 1) to assess computational speed. We found that for strands with 50 knots points, the path update for each simulation step typically required less than 10 iterations and a total compute time of 1 msec or less on an Intel core i7 desktop computer.

At TGCS 2019, we will show our technique integrated into several biomechanical models.

### REFERENCES


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