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A ROBUST OPTIMAL CONTROL METHOD TO ASSESS THE IMPACT OF SENSORIMOTOR NOISE ON HUMAN MOTION

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INTRODUCTION

Optimal control methods enable model-based predictive simulations of human movement to reveal how changes in the neuro-musculoskeletal system affect behavior. A feature of human motor control that is often not considered is robustness against sensorimotor noise present in the nervous and muscular systems. An optimal control framework that accounts for the presence of sensorimotor noise might allow answering questions related to movement variability and postural control in the presence of noise [1].

Robust optimal control methods consider uncertainty in the dynamics and cost function. Due to the increased numerical complexity of modeling uncertainty, applications of robust optimal control to simulate human movement have thus far been limited to models with linear dynamics [2]. However, human movement is best described by non-linear dynamics.

Here, we applied a robust optimal control method developed by Houska et al. [3] to simulate human movement based on a skeletal model governed by non-linear dynamics. Houska's method accounts for uncertainty by simulating the state co-variance matrix based on a linear approximation of the state co-variance dynamics along with the states [3]. The main advantage of this method over other methods that allow incorporating uncertainty in the dynamics (eg iLQG [4]) is its compatibility with direct collocation, an efficient method to simulate optimal human movement [5]. We implemented this method to simulate a squatting motion in the presence of motor noise and evaluated the effect of accounting for noise by comparing robust to nominal optimal control.

METHODS

Optimal control problems are generally written in the form of a constrained optimization:

$$\begin{aligned} &Minimise \;\; :: \;\; J\big(x(t),u(t)\big) \\ &subject\ to \;\; :: \;\; \dot{x}(t) = f\big(x(t),u(t)\big) \qquad (1) \\ &\qquad\qquad\qquad h\big(x(t),u(t)\big) \leq 0 \end{aligned}$$

with $x(t)$ the state trajectory, $u(t)$ the control trajectory, $J\big(x(t),u(t)\big)$ the cost functional describing the optimality principle (e.g. minimization of effort), $\dot{x}(t) = f\big(x(t),u(t)\big)$ the first-order differential equations describing the system dynamics and $h\big(x(t),u(t)\big) \leq 0$ path constraints and/or boundary conditions.

When introducing Gaussian noise, system dynamics are a function of the states, the controls and the variance w: $\dot{x}(t) = f(x(t),u(t),w)$. To account for the uncertainty in the dynamics, we augment the state-space by the state co-variance matrix $P(t)$ and describe the propagation of the co-variance matrix using the continuous Lyaponuv equation [3]. The new optimal control problem is larger but has the same form as (1):

$$\begin{aligned} &Minimise \;\; :: \;\; J(x(t),u(t),P(t)) \\ &s.t. \;\; :: \;\; \dot{x}(t) = f(x(t),u(t),0) \\ &\qquad \dot{P}(t) = A(t)P(t) + P(t)A(t)^T + C(t)\textstyle\sum_w C(t)^T \\ &\qquad h(x(t),u(t),P(t)) \leq 0 \\ &with \\ &A(t) :: \frac{\partial f}{\partial x}(x(t),u(t),w) \;, C(t) :: \frac{\partial f}{\partial w}(x(t),u(t),w). \end{aligned}$$

$\sum_w$ is a diagonal matrix with variances of the different noise sources on the diagonal. This augmented dynamic optimization problem can be transcribed as a non-linear programming problem (NLP) using direct collocation and solved efficiently with a gradient-based method whenever $J(.), f(.)$ and $h(.)$ are continuously differentiable with respect to x, u and w.

We implemented both the nominal and robust optimal control methods described above to simulate a squatting motion. Skeleton dynamics was described based on a sagittal plane model including an ankle, knee and hip degree of freedom. Each joint is actuated by an ideal torque actuator with maximal torque output of 150, 250, 200Nm respectively. Joint torques were modeled by linear time-variant feedback from the states (joint angles, velocities and actuator activations) and were subject to activation dynamics with a time delay of 80ms. In the robust optimal control problem, motor noise was modelled as Gaussian noise with a standard deviation of 1% of the maximal torques added to the actuator torques. The task was described by constraining the initial (t = 0s) and final (t = 1s) positions to be fully upright. At t = 0.4s the center-of-mass was constrained to be 0.275m lower than in the upright position. The center-of-pressure was constrained to be within a realistic base-of-support (-0.05m to 0.18m relative to the ankle joint). The cost J was a weighted sum of the time integral of the sum of squared joint torques and a regularization term for the feedback gains. In the robust optimal control problem, this function is augmented with the time integral of the variance of the states (diagonal elements of the co-variance matrix P). In both cases, we solved for 27 control gains (9 states x 3 actuated degrees of freedom) that minimized J. The problems were formulated as NLPs in CasADi [6] using a Radau collocation scheme with 100 mesh intervals and 3 collocation points per mesh interval. The resulting NLPs were solved with IPOPT [7].

RESULTS AND DISCUSSION

The nominal optimal control gains produced a squat motion in absence of motor noise (Fig.1 – blue). However, the nominal control law is not robust against 1% motor noise added to the actuating torques (Fig.1 – green). In contrast, the robust control law yields stable movements when 5% motor noise is added (Fig.1 – yellow/red).

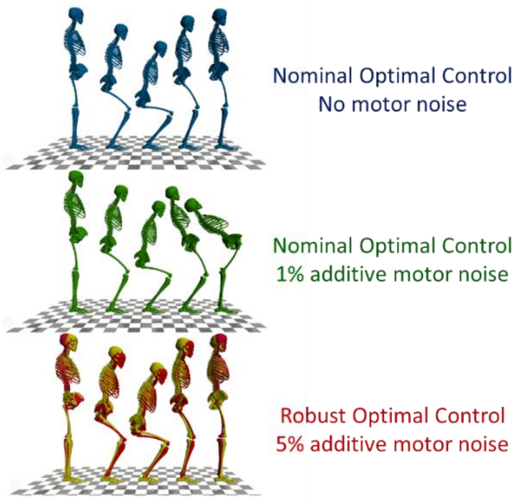

Fig 1: Minimal effort squat motion. The robust optimal control law is robust against motor noise.

Mean kinematics and joint torques of the robust and the nominal optimal controller are nearly identical (Fig. 2). However, the robust and optimal feedback gains are very different. The mean change in feedback gains averaged over time was 5.42 rad^{-1} for the position feedback gains, 1.77 s.rad^{-1} for the velocity feedback gains and 3.88 for the activation feedback gains.

There was no trade-off between robustness and the other part of the objective, i.e. effort minimization. Two model choices explain this. First, our time-variant full-state control law is highly redundant and this redundancy can be exploited to improve robustness without altering the optimal movement. Second, we did not model sensory noise (no noise added to the state in feedback law), allowing for (unrealistically) accurate state feedback.

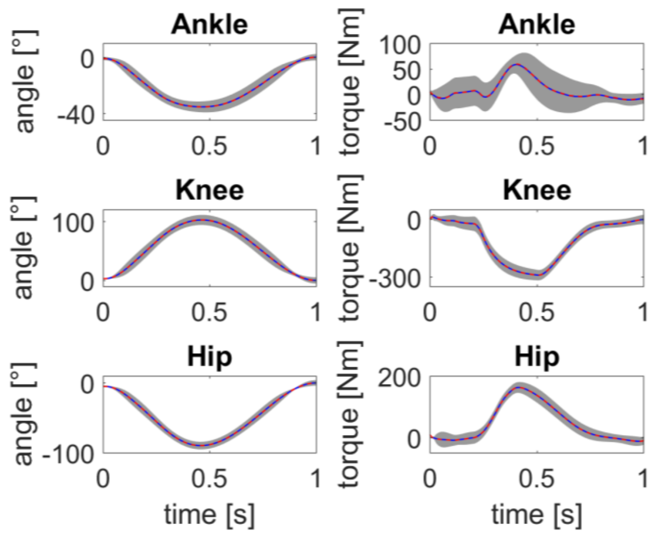

Fig 2: Nominal OCP (red dotted); Robust OCP (blue) - Joint positions and torques with 95% confidences intervals (for the robust OCP formulation only) derived from the co-variance matrix under 5% motor noise

CONCLUSIONS

Accounting for uncertainty due to noise, allowed us to find a feedback law that improved the robustness of a squat movement against motor noise. This example shows that a sufficiently complex feedback law can stabilize movement without increasing effort. Note that the same feedback law with gains that were not optimized for robustness led to unrealistic movements in the presence of noise.

A framework that accounts for movement variability due to sensorimotor noise is important to study motor control. Our example demonstrates that failing to account for noise results in unrealistic control laws.

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